

User-Guided Outlier Detection in Heterogeneous Datasets

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ABSTRACT

Rapidly developing areas of information technology are generating massive amounts of data. Human errors, sensor failures, and other unforeseen circumstances unfortunately tend to undermine the quality and consistency of these datasets by introducing outliers – data points that exhibit surprising behavior when compared to the rest of the data. Characterizing, locating, and in some cases eliminating these outliers offers interesting insight about the data under scrutiny and reinforces the confidence that one may have in conclusions drawn from otherwise noisy datasets.

In this paper, we describe a new user-guided outlier detection framework, *dBoost*, which relies on inference and statistical modeling of heterogeneous data to flag suspicious fields in database tuples. At the heart of the system lies a tuple expansion procedure, which reconstructs rich information from semantically poor SQL data types such as strings, integers, and floating point numbers. We show that this novel approach achieves good classification performance, both in traditional numerical datasets and in highly non-numerical contexts such as mostly textual datasets. Our implementation is publicly available, under version 3 of the GNU General Public License.

1. INTRODUCTION

Sensor glitches, submission of incorrect input, and malicious activities are a few examples of events that can lead to the appearance of outliers in a dataset. If undetected, these values can skew statistics, support invalid conclusions, slow database operations, and cause otherwise avoidable expenses. On the other hand, careful analysis of these values can yield new insight about the data, prevent undesirable events, and generally improve the reliability of the data [1].

Previous literature has generally defined an outlier as “an observation which deviates so much from the other observations as to arouse suspicions that it was generated by a different mechanism” [9], and has suggested a number of

ways to detect and in some cases eliminate suspicious values. Previous approaches to outlier detection include modeling numerical data using Gaussian Mixture Models [15, 17, 18], Histogram modeling [8, 20], and k -nearest neighbors [16].

Little work, however, has focused on developing generic methods for user-guided outlier detection that provide sufficient flexibility to offer insight on widely diverse and heterogeneous data stored in typical relational database management systems. The relative inexpressivity of basic SQL types – integers, strings, and doubles in particular – might be to blame: strings, for example, can be used to store information as diverse as city names, email addresses, or phone numbers; the task rests on application logic to parse these values in and out of the databases. This paucity of semantic information leaves outlier detection algorithms with very little information to work with.

This paper presents an innovative approach to reporting and flagging outliers in highly heterogeneous datasets: our tools systematically expand semantically poor SQL types to derive richer information, using this derived information as additional dimensions that can be used to detect outliers and provide detailed reports to the user. Expansion proceeds by systematically applying a set of type-dependent rules, which map values of one type to a set of derived attributes (all integers, for example, are expanded into dates and times by considering them as Unix timestamps, and into sets of booleans by considering them as bit vectors). These derived attributes often show more structure than the original values, allowing our models to efficiently learn soft constraints about the data. For example, if most of the values in a string column are in title case, then our models derive a soft constraint about the proper casing of values of that column, and report records with incorrectly formatted values. Similarly, if most of the dates reconstructed from an integer column fall on the same day of the week, then values falling on other days are flagged as suspicious. Expanded data can be analyzed further to detect soft functional dependencies between derived attributes, enabling the detection by multidimensional models of a broad class of data inconsistencies.

We designed the system to be both fast and memory efficient; it proceeds in three online passes over the data, keeping no more information than strictly necessary (in general, no more than a few dozen values per field in the database schema). The architecture is parallelizable, the analysis can be distributed over multiple computation nodes, and information can be kept from one run to the next so as to eliminate the first and possibly the second pass.

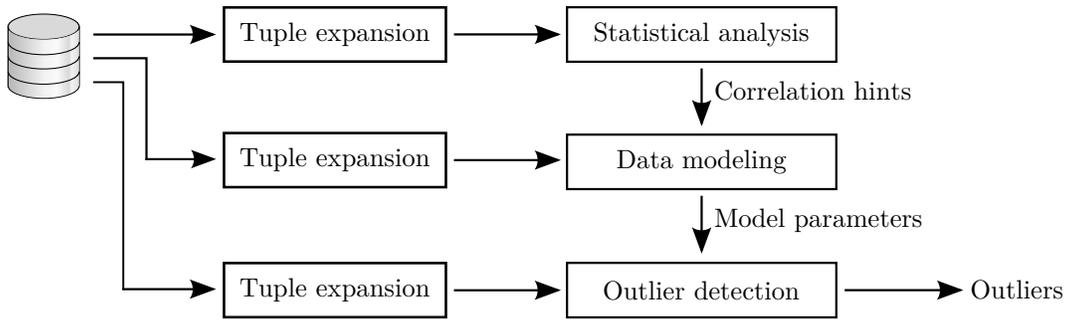


Figure 1: The *dBoost* pipeline

Our contributions are as follows:

1. We discuss the idea of tuple expansion and a set of rules that prove useful in flagging outliers in heterogeneous datasets.
2. We propose a histogram-based approach to detect outliers in non-numerical, heterogeneous datasets.
3. We evaluate the performance of our system on several synthetic and real-world datasets.

The rest of this paper is structured as follows: In Section 2 we give an overview of our system, and in Section 3 we discuss its technical aspects. We evaluate our tool on synthetic and real-world datasets in Section 4, and we describe related work in Section 5. Finally, we conclude in Section 6.

2. DBOOST OVERVIEW

We designed a framework that analyzes, models, and detects outliers in data. The whole system can be seen in Figure 1.

As a first step, tuples are read from the database and expanded: a set of type-dependent features is extracted for each field. These features express simple properties of the data, such as the length of a string, or the parity of an integer.

These expanded tuples are then analyzed in order to obtain simple statistical information, and to detect soft functional dependencies between different fields. The expanded tuples are then used to train one of three data models (Gaussian, Mixtures, or Histograms), with the help of the statistics and correlation hints gathered at the previous stage.

Finally, the trained model is used to classify tuples into regular records and outliers; these tuples can be the ones the model was trained with, or future inputs to the database system.

From a high level view, our pipeline is implemented as a three-pass streaming algorithm, requiring no memory beyond that required to train the individual models.

The different components of our system are summarized as follows and described in detail in the following sections:

1. Preprocessing – Tuples are expanded using knowledge about the database schema and field types (Section 3.1).
2. Statistical analysis – The expanded data is analysed to gather basic statistics, along with correlation information. These statistics are used for modeling and outlier detection (Section 3.2).

3. Data modeling – We apply various machine-learning algorithms to build models of the data (Section 3.3).
4. Outlier detection – Using the models built in the previous stage and user-provided sensitivity thresholds, we report outliers identified by the models trained during the previous stage (Section 3.4).

Table 1 illustrates these ideas on a very small dataset that includes a transaction ID, a registration date (Reg. date), and a social security number (SSN). We read the data row-by-row, and expand the registration date and SSN values into additional columns. The particular expansion rules are based on the type of each values: Reg. date is an INT, so it gets expanded, among others, into a year and a weekday. SSN is a STRING, and gets expanded into among others a length and a copy of the string with numbers stripped out and replaced by <num>. These tuple expansions are not materialized in the database, but rather fed into the model one-by-one, after which they are discarded.

3. FRAMEWORK IMPLEMENTATION

3.1 Preprocessing

Before being used to generate a model of the data, the tuples are expanded in order to reveal and extract potentially relevant information.

What if you need to store a date and time value with subsecond resolution? MySQL currently does not have an appropriate data type for this, but you can use your own storage format: you can use the BIGINT data type and store the value as a timestamp in microseconds, or you can use a DOUBLE and store the fractional part of the second after the decimal point.

High-Performance MySQL, 3rd edition (2012), p. 127

Data stored in databases often has rich semantics, encompassing dates and times, highly structured datatypes such as phone numbers, addresses, or GIS data. The semantics of plain SQL, however, are not expressive enough to properly store these rich data types. Programmers are thus forced to revert to simpler types, relying on application logic to parse the data.

Unfortunately, information on the structure of this data is also what an outlier detection engine would most benefit from. For example, the day of the week in a date may be relevant information in a banking application in which transactions

XID	Original Data			Expansions of Reg. Date (not materialized)			Expansions of SSN (not materialized)		
	Reg. Date	SSN		Year	Weekday	...	Length	Strip Numbers	...
1	1416497422	783-345-2351		2014	Thursday	...	12	<num>-<num>-<num>	...
2	1418201134	773-746		2014	Wednesday	...	8	<num>-<num>	...
3	1420359855	773-289-5552		2015	Sunday	...	12	<num>-<num>-<num>	...
4	1421575392	849-843-2729		2015	Sunday	...	12	<num>-<num>-<num>	...
5	01302015	773-387-9201		1970	Friday	...	12	<num>-<num>-<num>	...
6	1424866716	821-322-1857		2015	Wednesday	...	12	<num>-<num>-<num>	...
7	1425059692	822-971-1892		2015	Friday	...	12	<num>-<num>-<num>	...



Table 1: An example dataset showing outliers based on a histogram model. The rows detected as outliers are highlighted in red. The row with XID=2 is flagged due to its “Length” and “Strip Numbers” expansions: as seen in the corresponding histograms, the values 8 and <num>-<num> are seen few enough times in the data that they are flagged as suspicious. The row with XID=5 is also flagged as an outlier due to its incorrect registration date: indeed, the histogram analyzing the years shows that 1970 does not occur in the database frequently.

are only completed on weekdays, but unless the programmer explicitly duplicates this information in a separate column, it remains inaccessible to an automated outlier detector.

To reconstruct this lost information we expand tuples, breaking down each field into a sub-tuple of extracted type-dependent features. Figure 2 lists the rules that we apply to values of these three data types. These rules serve as a starting point for tuple expansion, but we expect users of our toolbox to expand this set of rules to add insight specific to their application domain. Expressing rules as simple functions mapping a value of a given type to a tuple of features makes it possible to express soft constraints about the data easily: instead of specifying hard constraints, users state that a certain way of looking at the data should be consistent across the rows of the table.

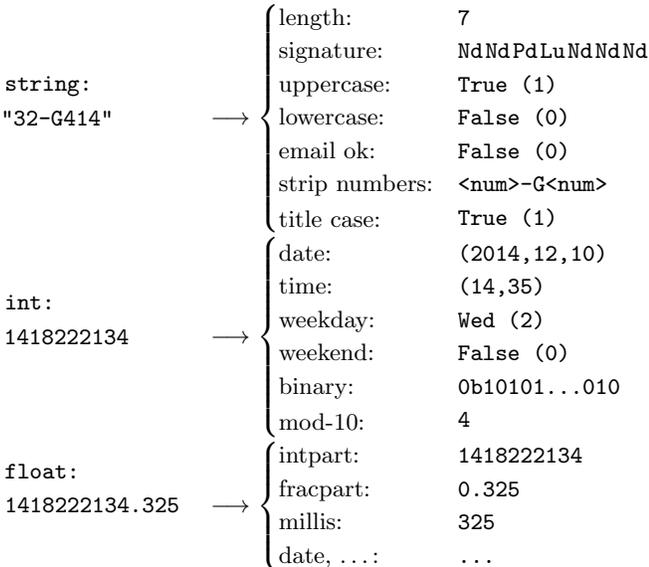


Figure 2: Selected tuple expansion rules.

The rules presented in Figure 2 are fairly general rules, likely to apply to be suitable to most datasets. For example, the `strip numbers` rule takes a string, and replaces

any sequence of digits by the string <num>. Such a rule is useful to check that a column is formatted consistently, allowing for variations only in the numbers: a database of legislative citations, for example, could contain entries such as S.933, H.R.21, P.L.107-155, or P.L.88-352; applying the `strip numbers` rule to this data yields S.<num>, H.R.<num>, P.L.<num>-<num>, or P.L.<num>-<num>; this reduces the whole dataset to a small number of patterns that can be easily analyzed using, for example, the histogram-based model of section 3.3.3.

The `signature` rule is equally interesting: to extract the signature of a string, our tools replace each character by the name of its Unicode class: uppercase Latin letters are Lu, lowercase Latin letters are Ll, digits are Nd, and punctuation signs are among others Po, Pe, Ps, and Pd. Such a rule is useful to check the consistency of various fixed-width formats such as ISO 8061 timestamps, birth dates, or phone numbers. As a more detailed example, a column in a database could be used to record references to particular sections, subsections, and paragraphs in a text: entries would be in the form §901.04(a), §853.02(d), or §910.45. Extracting the signature of these entries would yield PoNdNdNdPoNdNdPsLlPe for the first two entries, and PoNdNdNdPoNdNd for the last one. Just like before, this reduces the dataset to a small set of patterns, making it easy to detect discrepancies.

3.2 Statistical Analysis

Following the tuple expansion phase, the expanded tuples go through the statistical analysis phase. This phase collects simple statistics on each column of the table, and estimates which sets of columns are correlated.

The statistical information includes the average, variance, standard deviation and extremes of each numerical column, as well as approximate cardinality measurements for all columns. These statistics have two purposes. First, they are used at the end of the statistical analysis phase to determine which columns in the table are correlated. Second, the statistics can be used during data modeling to speed up the training phase of certain models by precompiling parameters needed by the models (e.g., mean and standard deviation for the Gaussian model).

We focus on two inter-column correlation strategies:

- For mostly-numerical datasets, we use Pearson’s product-moment correlation. It relies on the Pearson correlation coefficient, which measures linear dependencies between two vectors.

Given two column vectors X and Y , Pearson’s coefficient R is given by the following formula:

$$R = \frac{\text{Covar}(X, Y)}{\sqrt{\text{Var}(X) \text{Var}(Y)}} \quad (1)$$

R ’s value always lies between -1 and 1 . An R value close to 0 indicates little or no correlation, while values close to $+1$ or -1 indicate strong positive or negative correlations, respectively. Pairs of columns with a value of R above a user-specified threshold are added to a list of correlation hints, for use by the models.

- For mostly non-numerical datasets, we use a cardinality-based measure, flagging groups of expanded columns as correlated when their joint cardinality is below a user-specified threshold. When two columns are correlated (e.g., when one is computed directly from the other), the number of distinct pairs in the columns is similar to the number of distinct items in either column. On the other hand, when two columns are independent, the number of distinct pairs is close to the product of the number of distinct values in each column.

It is debatable whether correlations should be looked for in expanded tuple fields that come from the same original value. On the one hand such dependencies may provide valuable insight about the data (think for example of a date whose “day of week” value would correlate with the month, for example for an event happening all Mondays of May and all Thursdays of June). On the other hand, taking these subtuple correlations into account vastly increases the size of the search space, and may add spurious hits to the results. Experimentally, we found that discarding intra-field correlations made the entire process faster and more robust, and did not hurt accuracy on our test sets.

All aforementioned statistics and correlation hints can be computed using a single pass over the data: the expanded tuples are analyzed one row at a time, and the final statistics and correlations are computed after the last tuple has been processed. This contrasts with more advanced approaches to the detection of correlations and soft functional dependencies, such as the one used in CORDS [11]. Our simpler approaches yields a lower specificity, but still achieves good classification results, in part because each model only uses correlation hints as a guideline for interesting groups of columns to analyze. An excessive number of hints can thus affect performance, but does not significantly diminish the quality of the results. On the other hand, missing a correlation causes models to not analyze the corresponding group of columns, and thus to fail to uncover potential outliers. As with the other parts of our system, the correlation detector used in the statistical analysis phase is modular and could be replaced by any other scheme, including CORDS.

The results of the analysis pass are available to all models used at later stages in the tool.

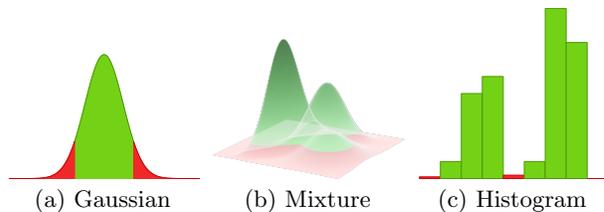


Figure 3: Simple visualization of the outlier detection strategy employed by each model

3.3 Data modeling

The current implementation of *dBoost* contains three data modeling strategies. Two of them are relatively simple machine-learning based applications that handle mostly continuous numerical data well; the last one makes much weaker assumptions about its input, and is best suited for studying discrete heterogeneous distributions produced by gathering human-input data. The following three subsections discuss the particulars of these three models; section 3.3.4 presents a way of extending models to deal with data that lends itself well to partitioning over a particular attribute.

3.3.1 Simple Gaussian Modeling

The univariate Gaussian model (Figure 3a) treats each value x_i of the expanded tuples as random sample drawn from a normal distribution $\mathcal{N}(\mu_i, \sigma_i)$.

The model’s parameters (a pair (μ, σ) for each numerical column) are computed as each column’s mean and standard deviation. In the common case where the dataset has not significantly changed between the analysis and the modeling passes, the information obtained during the statistical analysis pass is sufficient to derive these parameters.

Despite its simplicity, this model presents the attractive property of requiring extremely little memory – on the order of the size of one expanded tuple.

3.3.2 Mixture Modeling

The multivariate Gaussian Mixture model (Figure 3b) takes advantage of the correlation hints supplied by the statistical analysis pass to model sub-tuples of the expanded tuples as samples from multivariate Gaussian mixtures (GMMs), creating one model per group of correlated columns.

For example, if the statistical analysis phase outlines a pair of fields (f_1, f_2) as good candidate for joint modeling, then the Mixture modeling strategy will learn a particular GMM to model this correlation. Pairs of values (X_1, X_2) are here assumed to have been produced by random sampling off a distribution of the form

$$\sum_{j=1}^N \pi_j \mathcal{N}(\mu_j, \Sigma_j)$$

where N is the number of individual components that the GMM comprises (N is a user-defined value in our implementation, but abundant literature exists on the subject of choosing N [19] [3]), and π_j, μ_j and Σ_j are parameters of the GMM learned as part of the modeling pass [7].

Unlike simple Gaussian models, the expectation maximization algorithm used in inferring the optimal model parameters for Gaussian mixtures does require retaining some data in memory. Still, most of the fields obtained after expanding

each tuple are discarded after the relevant ones are extracted for learning purposes; in most cases we expect the set of values retained to be much smaller than the set of all attributes, thus limiting the memory usage.

In addition, when dealing with large amounts of data, it is possible – and indeed, preferable – to train the Mixture model on a randomly sampled subset of the data before running the full analysis. This approach is particularly relevant when using the Mixture model, but can be applied to all models to shorten the learning phase when dealing with very large datasets.

3.3.3 Histogram-based Modeling

Simple Gaussian modeling and Gaussian Mixture modeling both offer good results for continuous numerical data such as the ones produced by sensors, but suffer from two limitations:

- They make strong hypotheses about the distribution of the data under study
- They fail to capture patterns in discrete numerical data, non-numerical data, or heterogeneous data

Our last model (Figure 3c) does not make any assumption about the data under study. Instead, it counts the occurrences of each unique value in each column of the expanded tuple and for each set of potentially correlated sub-tuples (as suggested by the analysis module). These counts, accumulated over the entire dataset, provide a *de facto* distribution of the data in each field and set of correlated fields.

Described as above, the histogram modeling strategy is rather memory inefficient: it requires keeping track of each value of each expanded tuple that the model comes across. To limit memory usage, and to speed up the modeling phase, we instead discard histograms as soon as they reach a certain size – say, 16 bins. Discarding histograms when their number of bins reaches a fixed threshold is just one of a number of heuristics that could be implemented here; the idea is that a profusion of different values, all repeating infrequently, is unlikely to provide valuable insight as far as outlier detection is concerned (as an extreme example, the histogram of an attribute with no repeated values would only have one value per bin, and would not yield any insight about the data). With this discarding heuristic applied, histograms are quick to generate and extremely memory efficient.

Histograms also have the valuable property of treating sets of fields (obtained via correlation analysis) and single fields in the exact same way, thus permitting to model single columns or groups of attributes indifferently. Finally, because they make no assumption about the data they manipulate (aside from the requirement that it be of small cardinality), histograms are able to accurately describe a broad class of discrete distributions.

3.3.4 Meta-modeling through attribute-based partitioning

The models presented above treat attributes and sets of correlated attributes as a whole. In some cases, however, it is possible to identify sub-populations of tuples by scrutinizing certain expanded attributes of the data; these sub-populations can then be studied separately, yielding more insight and better outlier classification performance.

As an example, consider the case of an airline adjusting status levels for its frequent fliers, using the number of flights for

each passenger as well as their status level. A non-partitioned analysis may not return any interesting information, but a partitioned analysis could single out passengers in lower status levels traveling significantly more than average, or passengers with higher status traveling rarely. This would work even if statuses were stored as textual data, with no indication of their relative rankings.

The general approach, given a dataset and a pre-existing model, therefore consists in extracting sets of attributes based on correlation hints provided by earlier stages of the pipeline, and dividing each group of attributes between a single key (in the example, the status level) and one or more sub-population attributes (in the example, the number of flights). One instance of the selected model is then built for each value of the key. For example, if the statistical analysis phase highlights a correlation between columns A (status levels) and B (number of flights), and column A contains values a_1, \dots, a_n (**bronze**, **silver**, **gold**, ...), then we distribute the pairs (A, B) into n partitions based on the value of A ; values of B in each of these partitions are then modeled independently (in the example, this yields a different model of flights count for each status level).

This type of approach is useful when the distribution for an attribute or set of attributes is multi-modal. A high-level non-partitioned analysis will reveal values that fall in none of the classes; a partitioned approach, on the other hand, may more easily reveal discrepancies by suppressing interference between each class.

In addition to providing better classification accuracy, partitioning may lead to better runtime performance by diminishing the size of the dataset covered by each model. These benefits are especially important when model construction performance does not scale linearly, and when data volumes are too large to be analyzed on a single machine.

Finally, attribute-based partitioning allows for previously impossible analysis. Assuming for example that a dataset with two columns has 4 classes identified by the value in the first column, each with 10 distinct expected values in the second column, a generic histogram-based analysis would discard the histogram for the pair of values as having too many buckets (40). A partitioned analysis, on the other hand, would allow the construction of four histograms, each with 10 regular bins and potentially a few outliers.

In our prototype implementation, we focused on partitioning applied to the discrete histogram case; the technique, however, generalizes to all the models presented above.

3.4 Outlier Detection

Models, once properly trained, are used for classification and detection of outliers – either in incoming `INSERT` operations on a running system, or in existing rows (possibly but not necessarily the ones used during the model training phase).

Given that databases can contain tables with tens or hundreds of columns, simply flagging a row as an outlier is insufficient: users cannot be expected to painstakingly analyze each outlying row. Instead, *dBoost* automatically indicates which values in the row caused it to be flagged as an outlier.

The inter-column correlations are also taken into account during modeling: if the statistical analysis phase detected a correlation between two columns a and b , each tuple t will be augmented by an additional field that contains the corresponding pair (t_a, t_b) . This field is treated as a single,

multidimensional value, and is analyzed similarly to the other values by the models.

3.4.1 Simple Gaussian Modeling

The simple Gaussian model measures how much each value differs from the mean computed in the preceding pass. Given a tolerance parameter θ , a row is deemed an outlier if at least one of its attributes a has a value v_a such that

$$|v_a - \mu_a| \geq \theta \cdot \sigma_a \quad (2)$$

where μ_a and σ_a are the model’s parameters for column a , as described in Section 3.3.1.

In this model, detecting which values are responsible for the outlier flag is simply a matter of keeping track of which attributes satisfy Equation (2). The simple Gaussian model does not take correlation hints into account, and thus reports only single-attribute outliers.

3.4.2 Mixture Modeling

In the Mixture model, the likelihood of each (possibly multidimensional) field is evaluated using the corresponding GMM. This model operates under the assumption that data is accurately modeled by the chosen number of components in the GMM, and in particular that each non-outlying data point is well modeled by one of the Gaussians of the GMM.

This makes it possible to assign a Gaussian component to each tuple, and then flag as outliers the tuples that are not sufficiently well explained by their corresponding Gaussian (see [18]). Given a tuple t and its corresponding Gaussian c , this means rejecting t if

$$\pi_c \cdot \Pr(\mathbf{dist}(t, \mu_c) > d_0) \leq \theta \quad (3)$$

where θ is a user-defined parameter between 0 and 1, and d_0 is the Mahalanobis distance of t to the Gaussian.

As in the Gaussian Model, providing the user with a list of attributes that caused the row to be flagged as an outlier is simply a matter of tracking correlations that satisfied equation (3).

3.4.3 Histogram Modeling

The histogram-based modeling strategy proceeds in two phases to detect outliers.

First, after running through the learning phase, it decides for each histogram whether that histogram is “peaked” (i.e. showing a few strong modes) enough to be used to detect outliers. The aim of this phase is to discard histograms where most bins have a similar number of values, and are thus not useful for outlier detection. In practice, we use a simple statistical test to determine whether a histogram is sufficiently modal: we count the number of elements that fall into the most populated (“top”) bins, and discard the histogram if this count totals less than a user-specified proportion of values. Finding how many bins to include in the set of top bins is the most challenging part, and for this paper we explored two thresholding strategies (Figure 4):

- *Distribution-independent* – Given a histogram with N bins, we count only the values in the top bin if $1 \leq N \leq 3$, in the top 2 bins if $4 \leq N \leq 5$, and in the top 3 bins for $3 \leq N \leq 16$ (histograms with $N > 16$ bins were previously discarded). This method is stable when the set of bins is static (week days, booleans, . . .), but it is sensitive to the addition or removal of bins.

- *Distribution-dependent* – We sort the bins in increasing order of bin size b_i , and find the index i_{\max} such that the ratio $r = b_{i+1}/b_i$ is maximal (this calculation is safe, because the bin sizes are non-zero integers). If that ratio is under a user-defined threshold, we reject the histogram; otherwise, we consider bins $i_{\max}..end$ to be “top” bins.

Figure 4 shows various types of histograms, and lists the conclusions that each of these two approaches yield.

After identifying a relevant set of histograms (this operation only needs to run once, at the very beginning of the last pass), we proceed to the actual detection phase. We classify an expanded tuple X as an outlier if any of its values (or set of values, as grouped according to the correlation hints previously obtained) x_a verifies:

$$h_a(x_a) \leq \epsilon \sum_k h_a(k) \quad (4)$$

where $h_a(x)$ designates the number of tuples with value x for field a , and ϵ is a user-chosen sensitivity parameter.

In this model, identifying and reporting the outlying attributes is simply a matter of remembering which values x_a failed test (4).

3.4.4 Partition-based modeling

In the partition-based case, outliers are detected by the underlying models. To classify a given expanded tuple, each group of correlated attributes is divided between a one-attribute key and a group sub-population attributes. This group of attributes is then passed to the underlying model corresponding to the given value of the key, and the whole original tuple is reported as an outlier if any of its groups of sub-population attributes is marked as such by the underlying models.

4. EVALUATION

We implemented *dBoost* in version 3 of the Python programming language. Our code is publicly available on GitHub under version 3 of the GNU Public License¹. The program is made of two parts: a library, and a number of data acquisition front-ends (CSV and SQL are currently supported). The library provides functions for each of the phases previously described.

This sections presents the results of running our tool on the following real and synthetic datasets.

- *Synthetic datasets*

Fizz-Buzz A mixed textual-numerical dataset in which each record contains two entries: a number, and either “Fizz” if the number is divisible by 3, “Buzz” if the number is divisible by 5, “FizzBuzz” if it is divisible by both, and the number itself (as a string) otherwise. Outliers appear when the second column does not respect these rules; this can be a misplaced “Fizz”, a missing “Buzz”, or even a totally different string (e.g. “Woof!”).

Web logs A series of three non-numeric datasets in which entries contain the login time and connection location for different users. Each user has

¹<https://github.com/cpitclaudel/dBoost>

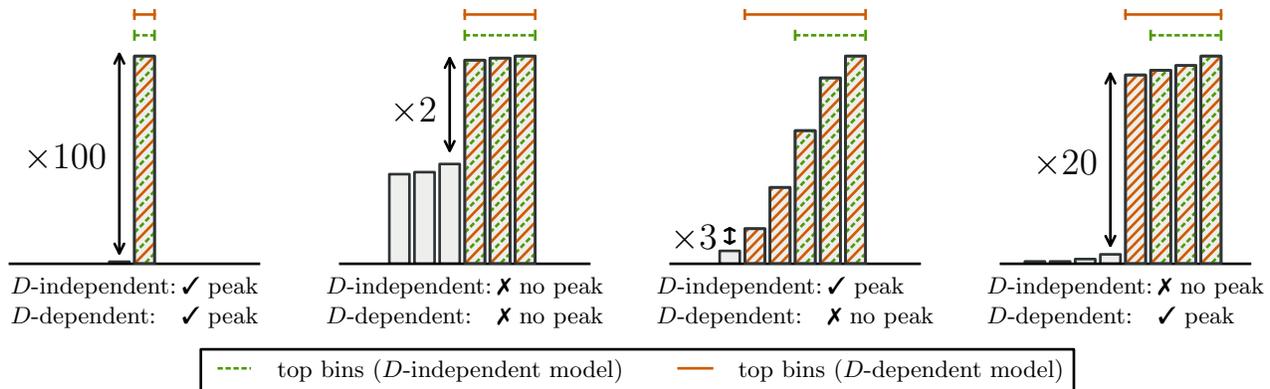


Figure 4: Sample histograms, and corresponding decisions with distribution-dependent (D -independent) and distribution-independent (D -dependent) thresholds. Each figure shows a sorted histogram, with the top bins hatched (in dotted green in the distribution-independent case, and in solid orange in the distribution-dependent case). The vertical arrows show the small value $r = 3$ in the distribution-dependent case. The weaknesses of the distribution-independent-model show in the third and fourth plots: in the third one the distribution-dependent strategy correctly rejects because of the small r ; in the fourth the distribution-independent strategy yields an incorrect threshold.

different connection habits, leading to different types of outliers.

- *Real-world datasets*

CSAIL Directory A publicly-accessible directory of researchers, in which each record may include a first and a last name, a phone number, an office number, an email, and a job title. Outliers are hard to define mathematically in this case, and we instead demonstrate how the ideas exposed in previous sections of the paper come together to allow for efficient detection of unusual values.

Intel lab data A publicly-available numerical dataset of temperature, light, humidity and voltage measurements. Outliers are due mostly to sensor glitches.

These datasets showcase the power of our methods, both in terms of classification power and expressiveness and succinctness when adding new rules to the system². Where relevant we include performance measurements. These numbers intend to demonstrate that our approach is computationally reasonable and that our models scale linearly given a fixed training size. We show in Section 4.5 how our prototype requires on the order of a few minutes to process a million elements using a high-level single-threaded scripting language. A production-ready implementation would run one or two orders of magnitude faster by taking advantage of the inherent parallelizability of the models, using an efficient on-disk representation of the data, and relying on a lower-level language with an efficient optimizing compiler.

The following subsections describe each of the test sets and associated results in greater detail.

4.1 Soft constraint specifications: Fizz-Buzz

We start with an extremely simple example, highlighting how easy our system makes it to encode and check data

²Indeed, the set of rules used for tuple expansion is user-configurable, and new rules can be easily added; thus, specific knowledge about the data can be taught to the system by users, expressing some soft form of data integrity constraints.

integrity constraints. The Fizz-Buzz programming exercise is based on a children’s game and frequently found in programming interviews. The synthetic dataset we generated obeys the following rules: for each record x, y , x is a number between 0 and 1000, and y is “Fizz” if $x \bmod 3 = 0$, “Buzz” if $x \bmod 5 = 0$, “FizzBuzz” if $x \bmod 15 = 0$, and x otherwise. In our synthetic dataset we introduced three outliers: (25, “Fizz”), (28, “Woof!”), (30, “Buzz”). Each demonstrates a different error, namely swapping “Fizz” and “Buzz”, producing entirely incorrect output, and failing to recognize that a number is divisible by both 3 and 5.

A traditional way of checking that all tuples verify the production rule outlined above is to encode this rule itself as a database integrity constraint. This requires encoding the full complexity of the exercise in the rule, and would require manual adjustments if the rules were to change. Instead, a user might want to specify the bare minimum for the system to infer the rules; in this case, it is sufficient to add one extraction rule, mapping integers to two booleans denoting whether they are divisible by 3 or 5. Such a rule could be written like this:

```
@rule
def fizzbuzz(x: int) -> ("div 3", "div 5"):
    return (x % 3 == 0, x % 5 == 0)
```

Running the discrete statistical analyzer on the synthetic datasets suggests that the two columns are correlated, and using the histogram model flags the aforementioned outliers. The output of the program for the (30, “Buzz”) line, for example, is similar to:

```
30 Buzz
> Values (30, 'Buzz') do not
  match features ('div 3', 'strip numbers')
• histogram for ('div 3', 'strip numbers'):
[532] ██████████ (False, '<num>')
[133] ██████ (False, 'Buzz')
[ 1] █ (False, 'Fizz')
[ 1] █ (False, 'Woof!')
[ 1] █ (True, 'Buzz')
[267] ██████████ (True, 'Fizz')
[ 66] ██████ (True, 'FizzBuzz')
```

Using the partitioned histogram model produces similar output:

```
30 Buzz
> Values (30, 'Buzz') do not
  match features ('div 3', 'strip numbers')
• histogram for ('strip numbers',) if 'div 3' = True:
  [ 1] 0 ('Buzz',)
  [267] ████████████████████ ('Fizz',)
  [ 66] ██████ ('FizzBuzz',)
... if 'div 3' = False:
  [532] ████████████████████ ('<num>',)
  [133] ██████ ('Buzz',)
  [ 1] 0 ('Fizz',)
  [ 1] 0 ('Woof!',)
```

4.2 Logins: a more realistic partitioned dataset

Our web activity synthetic datasets are comprised of two columns: a Unix timestamp stored as an INT, and a country. Each dataset is supposed to track the connections of a registered user on a website; such a dataset could be obtained by selecting the relevant rows out of a large table listing all connections of all users. Each user exhibits a different connection pattern:

- One user always connects from the same country; values that do not match this country are outliers.
- The second connects from one country during the week, and from another during the week-end; outliers in this case are connections from a country that doesn't match the country for that day of the week.
- The third user connects from a set of three countries, with no discernible pattern. This should not return any outliers.

The datasets are randomly generated sets of 2000 connections, listed in no particular order. The target outlier rate is 5% in each generated dataset.

As in the *Fizz-Buzz* example, numerical models are useless here, whereas histogram-based models produce good results. In the first case, a histogram-based model with no correlation analysis is sufficient to flag the outliers (analysis time: 0.08s, training time: 0.14s, total runtime: 0.38s)³. In the second case, the discrete statistical analysis phase singles out interesting pairs of correlated columns, including (*date#day of week, country*) and (*date#is weekend, country*). A histogram-based model is sufficient to successfully flag outliers, without resorting to partitioning.

Mixing data from two or more users, however, shows the limits of the non-partitioned histogram approach. If we only look at two-columns correlations the individual behavior patterns become less apparent, and if we look at three-column correlations the histograms become too large and spurious hits start to appear due to the many discrete correlations hints returned by the analyzer. The partitioned histograms model, on the other hand, can handle the three-users without particular difficulties, by highlighting (among others) the triplet (*user, date#is weekend, country*) (analysis time: 0.14s, training time: 0.19s, total runtime: 0.56s).

³All runtime results were obtained using a 4 core i7-4810MQ CPU @ 2.80GHz and 32GB of RAM.

4.3 CSAIL Directory

The CSAIL directory is an online directory of about 1000 faculty, staff and students in the MIT Computer Science and Artificial Intelligence Laboratory⁴. Each entry contains a person's name, phone number, office number, email address, and position.

Some entries, such as a phone number, may be missing from the directory. Nonetheless, we expect our framework to be useful in flagging discrepancies between different records. Since the notion of what constitutes an outlier here is imprecise at best, we also expect the tool to allow the user to explore different sets of parameters. To illustrate the process, we present the results returned by two iterations of the tool in the next subsection, each with increasingly strict limits on the number of outliers returned. Because the CSAIL test set is exclusively textual, we use the histogram model for evaluation; continuous models would not fare as well, since only part of the expanded tuples are numeric.

4.3.1 Initial run: low specificity filtering

The search for outliers is initiated with parameters $\theta = 0.8, \epsilon = 0.2$ (analysis time: 0.36 s, training time: 0.11 s, total runtime: 0.60 s). Correlation detection is disabled for these experiments.

This invocation produces a long list of outliers; a small subset of these is presented below. For privacy reasons, names, phone numbers, office numbers, and emails have been omitted or anonymized in the following listings.

```
Hacker, Alyssa, 32-D968,
  aph@CSAIL.MIT.EDU, Postdoctoral Associate
> Value 'aph@CSAIL.MIT.EDU' doesn't match feature '
  lower case'
```

```
Bitdiddle, Ben, NE47-989,
  bbitdid@mit.edu, Graduate Student
> Value 'NE47-989' doesn't match feature 'signature'
```

```
Lu-ater, Eva, 32-G972,
  eva@csail.mit.edu, Research Scientist
> Value 'Lu-ater' doesn't match feature 'title case'
```

```
Tweakit, __, 32-G699,
  twktem@mit.edu, Administrative Assistant
> Value '_' doesn't match feature 'empty'
```

In total, 451 entries contain outliers, out of a total of 1000. Office numbers are often flagged, as well as names and email addresses. By changing the input parameters to $\theta = 0.8, \epsilon = 0.05$, most of the outliers due to office numbers disappear due to the lower sensitivity. Hacker, Alyssa disappears from the list, since e-mails with inconsistent capitalization occur frequently enough in the database that they are not considered outliers at sensitivity level $\epsilon = 0.05$. After tuning these parameters, we are left with 68 outliers.

In addition to identifying outliers, *dBoost* is equipped with tools that provide the user with additional feedback on why features were identified as outliers.

```
Bitdiddle, Ben, NE47-989,
  bbitdid@csail.mit.edu, Graduate Student
> Value 'NE47-223' doesn't match feature 'signature'
• histogram for ('signature',):
  [266] ████████████████████ <empty>
  [ 1] 0 Lu,Lu,Nd,Nd,Pd,Nd,Nd,Nd
  [ 1] 0 Lu,Nd,Nd,Pd,Nd,Nd,Nd
```

⁴<https://www.csail.mit.edu/peoplesearch>

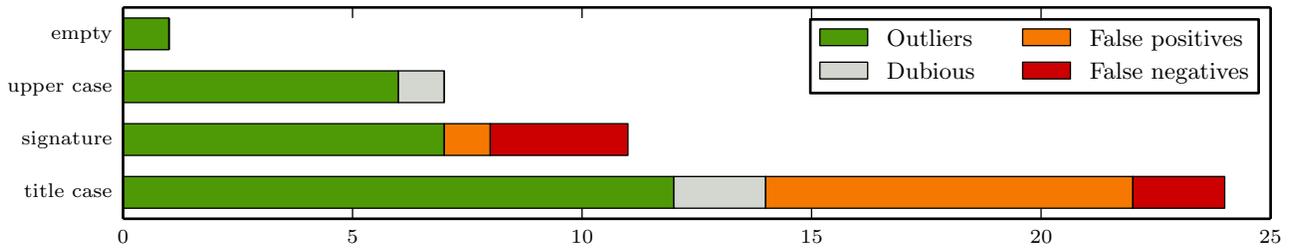


Figure 5: Accuracy of *dBoost* on the CSAIL dataset, evaluated by comparison to manual annotation of outliers in the directory. Outliers were detected using $\theta = 0.8$ and $\epsilon = 0.025$; the tuples are sorted according to why they are – or are flagged as – outliers. The false positives in the last category are due to names whose proper capitalization is not title case. Green represents region of agreement between the output of *dBoost* and manual annotation, red shows outliers in the manual annotation that *dBoost* did not find, orange shows records that *dBoost* flagged as outliers that the manual annotation did not, and grey indicates values that could not manually be categorized with complete certainty as either outlying or non-outlying.

```
[ 2] 0 Nd,Nd,Lu,Pd,Nd,Nd,Nd
[485] //Nd,Nd,Pd,Lu,Nd,Nd,Nd
[ 51] //Nd,Nd,Pd,Lu,Nd,Nd,Nd,Lu
[155] //Nd,Nd,Pd,Nd,Nd,Nd
[ 36] //Nd,Nd,Pd,Nd,Nd,Nd,Lu
[  3] 0 Nd,Nd,Pd,Nd,Nd,Nd,Nd
[  1] 0 Nd,Pd,Nd,Nd,Nd
```

```
Lu-ater, Eva, 32-G972,
eva@csail.mit.edu, Research Scientist
> Value 'Lu-ater' doesn't match feature 'title case'
• histogram for ('title case',):
[ 15] 0 False
[986] //True
```

```
Tweakit, _, 32-G699,
twktem@mit.edu, Administrative Assistant ...
> Value '_' doesn't match feature 'empty'
• histogram for ('empty',):
[1000] //False
[  1] 0 True
```

Our tool highlights the incorrect field, and prints the corresponding histogram. The bin in which the suspicious value falls is also highlighted. The `signature` case is particularly interesting: recall that to extract the signature of a string, our tools replace each character by the name of its Unicode class; hence the string `NE47-989` is converted to `Nd,Nd,Nd,Nd,Nd,Nd,Nd,Nd` (two letters, two numbers, one dash, three numbers), which does not fall in any of the dominant bins (the most frequent case, `Nd,Nd,Pd,Lu,Nd,Nd,Nd`, describes office numbers like `32-G804`, the predominant form of office numbering in the Stata Center).

Manual inspection of the results reveal that most of the outliers reported are actually bad inputs. There are, however, a number of false positives, such as:

```
DeFect, Cy, 32-D597,
cydf@csail.mit.edu, Graduate Student
> Value 'DeFect' doesn't match feature 'title case'
• histogram for ('title case',):
[ 15] 0 False
[986] //True
```

The case of `DeFect` is correct, but our tool notes that it does not adhere to the casing standard derived from other tuples, and thus reports it.

We compared *dBoost*'s output to a manually annotated version of the CSAIL directory to analyze its accuracy; the results are shown in Figure 5.

4.4 Intel Lab Data

We also evaluated our outlier detection framework on sensor data from the publicly available Intel Lab Data set⁵. The Intel Lab Data contains data collected from 54 sensors spread throughout the Intel Berkeley Research Lab. Each data entry contains information including temperature, humidity, light and voltage taken from a Mica2Dot sensor and weatherboard. The dataset contains a total of approximately 2.3 million measurements.

The Intel lab dataset has known outliers from faulty sensor readings due to periods of critically low voltage. During these periods, the sensors go haywire and produce faulty measurements. For example, the temperature may be registered as over 120 degrees Celsius, which is obviously abnormal behavior in a human environment such as where the sensors were deployed.

We analyzed a sample of 1000 data points selected at random from the sensor data; due to the numerical nature of this data, the Simple Gaussian and Mixture models are better-suited to analyzing it than the Histogram model.

We also compare the results of our models to Local Outlier Factors, a common outlier detection methodology, in this section.

4.4.1 Simple Gaussian Model

The results from running the sensor data set through the Simple Gaussian model are shown in Figure 7a. The data is plotted in light green, and the outliers are marked by dark red crosses.

In this experiment we flag the entries with column values that fall outside 1.5 standard deviations of the mean of that particular column as outliers. This model runs relatively fast, as no correlations are computed (analysis time: 0.03 s, training time: 0 s, total runtime: 0.12 s).

4.4.2 Mixture Model

We set the statistical threshold to 0.7, which produces two correlations between temperature and humidity and between temperature and voltage. Figure 7b shows the results when using a single Gaussian component. Points flagged as outliers have a likelihood of less than 7.5% of being produced by the Gaussian generated by the model (analysis time: 0.03 s, training time: 0.34 s, total runtime: 0.73 s).

This model is able to detect values with high temperature and low voltage as outliers.

⁵<http://db.csail.mit.edu/labdata/labdata.html>

Figure 7c shows the results obtained using the Mixture model with two components, using the same 1000 randomly selected data points. Flagged values have a likelihood of less than 7.5% under their dominant Gaussian (analysis time: 0.03s, training time: 0.35s, total runtime: 0.78s). When using two Gaussians, the points clustered around the temperature 120 degrees Celsius are no longer detected as outliers: they are modeled by their own Gaussian (although this Gaussian’s weight is smaller than its counterpart). This model highlights the points within normal sensor operation that have outlying results.

4.4.3 Local Outlier Factors

In this section we compare the results of our Gaussian and Mixture models to Local Outlier Factors (LOF) [5], a frequently used method for outlier detection. LOF measures the degree to which a data point is an outlier by comparing each data point’s reachability to those of its k nearest neighbors. The higher the LOF, the more isolated the data point relative to its local neighborhood and therefore the more likely the point is to be an outlier.

One downside of LOF compared to $dBoost$ is that it can only evaluate two-dimensional data. The original algorithm also has significant computation complexity in order to calculate the distance to the nearest neighbors of each data point. One benefit of LOF, however, is that the algorithm returns a continuous value that indicates the degree to which a point is an outlier, as opposed to a binary value.

Figure 7d shows the outliers detected by LOF when $k = 2$. We observe that contrary to the Gaussian and Mixture models, the outliers detected by LOF are scattered throughout the data. The outliers are not necessarily the points one would intuitively assume are outliers. This is because points that are within the normal range of the data will be selected as outliers if they are far enough away from the other points nearest to them. We find that LOF is not as useful at pointing out the tagged outliers in the sensor data set.

4.5 Scalability

We measure the total runtime of our system, including the data modeling and outlier detection phases for the Simple Gaussian, Mixtures with 2 Gaussians, and Histograms. We used the Intel sensor data set from Section 4.4 to evaluate the Gaussian and Mixture models, and the CSAIL directory from Section 4.3 to evaluate the Histograms. We use random sampling to provide training sets of 1 thousand and 10 thousand elements from the Intel dataset to build the data models. We test them on all 2+ million elements in the dataset. To provide a more comprehensive study of the scalability of the Histogram model, we replicated the rows of the CSAIL directory to increase the training and test set sizes.

In Figure 6, we show the runtime of our prototype. Each line shows a model trained with a different training set size. As shown in the figure, runtime scales linearly as the test set size increases. Applying additional optimizations such as using a lower-level language or enabling parallelism would improve runtime performance to production-ready levels.

5. RELATED WORK

There has been substantial research in how to build models to detect outliers [2], including how to detect outliers in high-dimensional data by searching the subspaces of the data

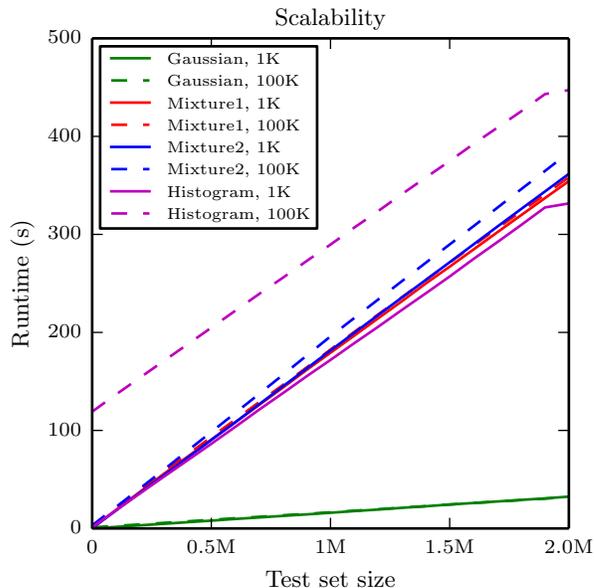


Figure 6: The scalability of the Gaussian and Mixture Models produced with different training sample sizes (listed next to the model in the legend) as the test set size increases.

[23][12]. However many of these algorithms are complex and can require substantial computation to determine whether a new data point lies outside the data.

Several algorithms exist in the data mining community to determine outliers when doing data analysis. Local outlier factor measures the degree to which a data point is an outlier [5]. Other techniques include k-nearest neighbor [16] and cluster analysis.

Research has been done to attempt to explain why outliers exist given properties of the original data [22]. Unlike our tool, Scorpion starts with user-defined outliers and works backwards to find potential explanations as to why the data points are outliers.

Statistical methods have been used to detect dependencies between columns of relational databases for the purpose of informing the query optimizer of potential data dependencies [11]. These methods require only a small sample of the data to detect functional dependencies with high probability of correctness. The relatively low computation required by these algorithms makes them more amenable to detecting data anomalies in real time. However, these methods are better suited for numerical data [10]

Gaussian Mixture Models have been used for outlier detection in multiple contexts [15, 17, 18].

Histograms are used in conjunction with local outlier factors to detect outliers [8, 20], in cases of numerical or categorical data.

To the extent of our knowledge, the literature regarding outlier detection on non-numerical data is much less extensive. Some common approaches include identifying outliers using a similarity measure [6], Probabilistic Suffix Trees [21] and sequence alignment [4].

Some specialized work has focused on inferring domain-specific rules on highly specific data such as a sequence of UNIX commands [13, 14]. By contrast, we take on a general-purpose approach that is capable of dealing with data as diverse as a set of names and office numbers to real-valued

sensor data. Additionally, we analyze data without any additional information on its structure.

Overall, we differ from previous approaches in that we are capable of analyzing a very wide range of data and do not use predefined rules for outlier detection – although adding user-defined rules is possible in our framework.

6. CONCLUSION

In this paper we presented *dBoost*, a toolkit that leverages tuple expansion to detect outliers in both numerical and heterogeneous data sets. We demonstrated that well-known machine-learning strategies could be used to flag spurious numerical and to a lesser extent non-numerical data. We also demonstrated that simple correlation modeling is useful in inferring data dependencies and improving the accuracy of outlier detection procedures. We discussed histogram-based models, and showed that they provided a useful tool in analyzing mostly textual data.

We showed that our toolkit performs well on real-world problems, including identifying potentially wrong entries in a people directory and flagging erroneous values generated by faulty sensors. Our toolkit and its source code are available for public use under a permissive license, with the hope of allowing database users to formulate their own type-based rules and find discrepancies in their own data.

We believe that the preliminary results presented in this paper are promising, especially in the area of identifying outliers in heterogeneous data.

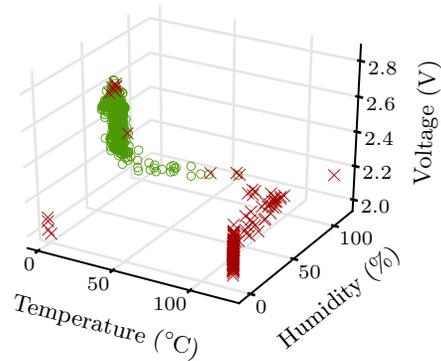
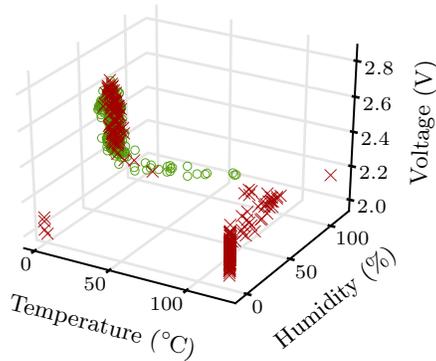
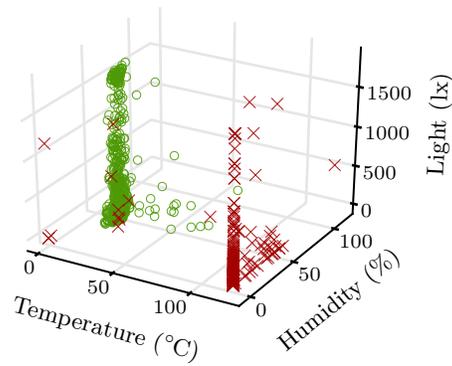
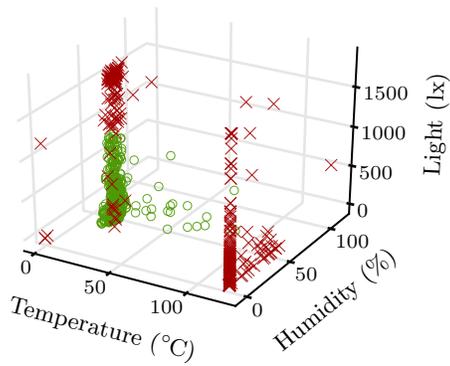
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8. ADDITIONAL AUTHORS

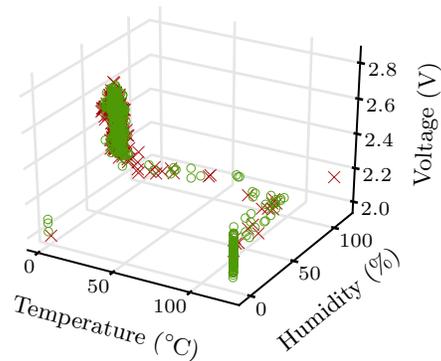
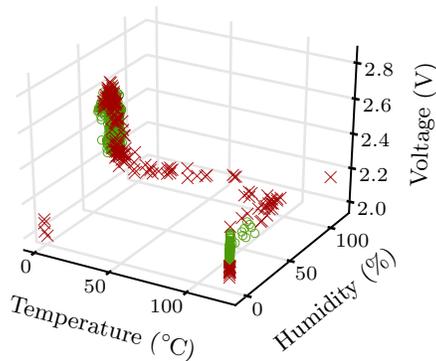
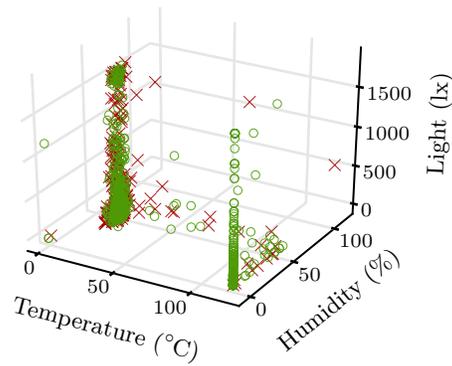
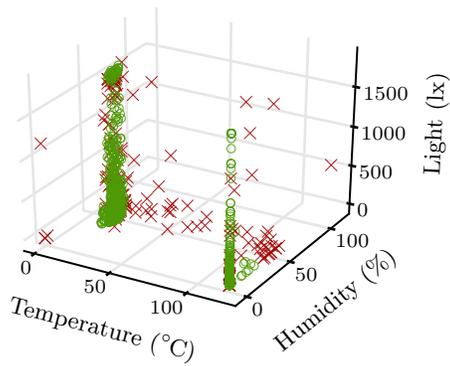
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(a) Simple Gaussian, $\theta = 1.5$

(b) GMM, $n = 1$, $\theta = 0.1$



(c) GMM, $n = 2$, $\theta = 0.05$

(d) Local Outlier Factor, $k = 2$

Figure 7: A comparison of various modeling approach on the Intel dataset. Red crosses indicate detected outliers. Figure 7a shows the results of simple uni-dimensional Gaussian modeling (Subsection 3.3.1). Figure 7b shows what happens when correlations are taken into account, using a multidimensional GMM with a single Gaussian (Subsection 3.3.2). Figure 7c shows the results of using two Gaussians instead of one to model the data. Note how the secondary data blobs, detected as outliers in the first two cases, are in the third case identified as valid data. Figure 7d shows the results of the Local Outlier Factor Algorithm on the same dataset.